```
> library(daewr)
> data(cakeb)
> library(lme4)
> mmod <- lmer(y ~ x1 + x2 + x1:x2 + x1sq + x2sq + (1|Ovenrun),
+ data = cakeb)
```

A comparison of the parameter estimates obtained by the least squares method and REML is shown in Table 10.11. Here it can be seen that the estimated linear main effects are the same for both methods, but the standard errors are much smaller for the sub-plot effect and larger for the whole-plot effect using the more correct REML method. The other estimates differ and again the tests for the sub-plot effects are more sensitive using REML and the tests for whole-plot effects are less sensitive. The REML estimated whole-plot and sub-plot error variances are shown at the bottom on the right side of the table. Letsinger *et al.* (1996) showed that when $\sigma_\omega^2/\sigma^2 < 0.25$ the least squares estimates will be reasonably close to the REML estimates and can be used in practice. However, since σ_ω^2/σ^2 is usually not known, the REML method should be used first.

Table 10.11 *Comparison of Least Squares and REML Estimates for Split-Plot Response Surface Experiment*

	Least Squares (`rsm`) Func.			REML (`lmer`) Func.		
Factor	$\hat{\beta}$	$s_{\hat{\beta}}$	P-Value	$\hat{\beta}$	$s_{\hat{\beta}}$	P-Value
intercept	2.979	0.1000	<.001	3.1312	0.2667	0.054
x_1	−0.2500	0.0795	0.026	−0.2500	0.2656	0.399
x_2	−0.4333	0.0795	0.003	−0.4333	0.0204	<.001
x_1^2	−0.6974	0.1223	0.002	−0.6835	0.3758	0.143
x_2^2	0.1526	0.1223	0.016	−0.0965	0.0432	0.089
x_1x_2	−0.3500	0.0973	0.268	−0.3500	0.0250	< .001
				$\hat{\sigma}_\omega^2 = 0.1402$, $\hat{\sigma}^2 = 0.0025$		

In some split-plot designs, the least squares estimators are identical to the estimators produced by REML. These designs have an advantage in that the parameters estimates do not depend on the variance components in the split plot design. From a practical point of view, it would be advantageous to be able to use the least squares estimates in R, since the `rsm` function, that uses the least squares method, can also automatically compute the canonical analysis, the ridge analysis, or produce predicted values over a grid for use in contour plotting. The `lmer` function, that produces the REML estimators, does not produce these analysis that are useful for response surface exploration.

Vining *et al.* (2005) proved an equivalence theorem that shows the least squares estimates of all of the regression coefficients in a split-plot central